

Do Quarks Obey D-Brane Dynamics?

Amir H. Fatollahi

*Institute for Studies in Theoretical Physics and Mathematics (IPM),
P.O.Box:19395-5531, Tehran, IRAN.*

fath@theory.ipm.ac.ir

Abstract

The potential between two D0-branes at rest is calculated to be a linear. Also the potential between two fast decaying D0-branes is found in agreement with phenomenology.

1 Introduction

In the last few years our understanding about string theory is changed dramatically; a stream which is called "second string revolution" [1]. The scope of this stream is presentation of a unified string theory as a fundamental theory of the known interactions. One of the most applicable tools in the above program are Dp -branes [2, 3]. It was conjectured, and is confirmed by various tests, that these objects can be considered as a perturbative representation of nonperturbative(BPS) charged solutions of the low energy of superstring theories.

On the other hand, the idea of string theoretic description of gauge theories is an old idea [4] [5]. Despite of the years that passed on this idea, it is also activating different research in theoretical physics [6] [7].

It has been known for long time that hadron-hadron scattering processes have two different behaviors depending on the amount of the momentum transfers [8]. At the large momentum transfers interactions appear as interactions between the hadron constituents, partons or quarks, and some qualitative similarities to electron-hadron scattering emerge. At high energies and small momentum transfers Regge trajectories are exchanged, the same which was the first motivation for the string picture of strong interactions¹. Besides the good fitting between Regge trajectories and the mass of strong bound states has remained unexplained yet.

Deducing the above partially different observations from a unified picture is attractive and it is tempting to search for the application of the recent string theoretic progresses in this area.

In this way one finds Dp -branes good tools. As the first step we try to extract some known results from the dynamics of Dp -branes. It is found that the potential between static D0-branes is a linear potential, the same which one expects in "cigar" model. Also the potential between two fast decaying D0-branes is calculated and the general results is found in agreement with phenomenology. Discussions are presented finally.

¹Regge behavior recently is used for fitting the experimental data with a considerable success [9]

2 On D-branes

D p -branes are p dimensional objects which are defined as (hyper)surfaces which can trap the ends of strings [3]. One of the most interesting aspects of D-brane dynamics appear in their *coincident limit*. In the case of coinciding N D p -branes in a (super)string theory, their dynamics is captured by a dimensionally reduced $U(N)$ (S)YM theory from (9)25+1 to $p+1$ dimensions of D p -brane world-volume [10, 3, 11]. Due to matrix models [12, 13] it has been understood that the supersymmetric gauge theories corresponding to $p = 0, -1$ contain many aspects which one expects from 11 and 10 dimensional supergravities (see e.g. [14]).

In case of D0-branes $p = 0$, the above dynamics reduces to quantum mechanics of matrices, because, only time exists in the world-line. The bosonic part of the corresponding Lagrangian is [12, 15]

$$L = m_0 \text{Tr} \left(\frac{1}{2} D_t X_i^2 - \frac{1}{(2\pi\alpha')^2} \mathcal{V}(X) \right), \quad i = 1, \dots, 9, \quad (1)$$

where $\frac{1}{2\pi\alpha'}$ is the tension of the fundamental string and $D_t = \partial_t - iA_0$ acts as covariant derivative in the 0+1 dimensional gauge theory. For N D0-branes X 's are in adjoint representation of $U(N)$ and have the usual expansion $X_i = x_{i(a)} T_{(a)}$, $a = 1, \dots, N^2$. The potential $\mathcal{V}(X)$ is

$$\mathcal{V}(X) = - [X_i, X_j]^2. \quad (2)$$

In fact (1) is the result of the truncation of the string theory calculations in the so-called "*gauge theory limit*" defined by

$$\begin{aligned} l_s &\rightarrow 0, \\ \frac{v}{l_s^2} &= \text{fixed}, \\ \frac{b}{l_s^2} &= \text{fixed}, \end{aligned} \quad (3)$$

which v and b are the relative velocities and distances relevant to the problem and l_s is the string length.

Firstly let us search for D0-branes in the above Lagrangian.

For each direction i , there are N^2 variables and not N which one expects for N particles. Although there is an ansatz for the equations of motion which restricts the $T_{(a)}$ basis to its N dimensional Cartan subalgebra. This ansatz causes vanishing the potential (2) and one finds the equations of motion for N free particles. In this case the $U(N)$ symmetry is broken to $U(1)^N$ and the interpretation of N remaining variables as the classical (relative) positions of N particles is meaningful. The centre of mass of D0-branes is represented by the trace of the X matrices.

In the case of unbroken gauge symmetry, the $N^2 - N$ non-Cartan elements have a stringy interpretation, governing the dynamics of low lying oscillations of strings stretched

between D0-branes. Although the gauge transformations mix the entries of matrices and the interpretation of positions for D0-branes remains obscure [14], but even in this case the centre of mass is meaningful. So the ambiguity about positions only comes back to the relative positions of D0-branes.

To calculate the effective potential between D0-branes one should find the effective action around a classical configuration. This work can be done by integrating over the quantum fluctuations in a path integral. For the diagonal classical configurations, which we know them as classical representation of D0-branes, the quantum fluctuations which must be integrated are the off diagonal entries. According to the above picture, this work is equivalent with integrating over the oscillations of the strings stretched between D-branes. Because here we are faced with a gauge theory, and our interests is the calculation around a classical field configurations, so it is convenient to use the background field method [16] for calculation the effective action.

For calculation the effective action we write (1) in D space-time dimensions in the form (in the units $2\pi\alpha' = 1$ and after the Wick rotation $t \rightarrow it$ and $A_0 \rightarrow -iA_0$)

$$\begin{aligned} L &= m_0 \text{Tr} \left(\frac{1}{4} [X_\mu, X_\nu]^2 \right), \quad \mu, \nu = 0, 1, \dots, D-1, \\ X_0 &= i\partial_t + A_0, \quad S = \int L dt, \end{aligned} \quad (4)$$

which μ and ν are summed with Euclidean metric. The one-loop effective action of (4) has been calculated several times (e.g. see the Appendix of [13] with ignoring the fermions) and the result can be expressed as

$$\left(\int dt \right) V(X_\mu^{cl}) = \frac{1}{2} \text{Tr} \log \left(P_\lambda^2 \delta_{\mu\nu} - 2iF_{\mu\nu} \right) - \text{Tr} \log \left(P_\lambda^2 \right), \quad (5)$$

which the second term is due to the ghosts associated with gauge symmetry and

$$P_\nu * = [X_\mu^{cl}, *], \quad F_{\mu\nu} * = [f_{\mu\nu}, *], \quad f_{\mu\nu} = i[X_\mu^{cl}, X_\nu^{cl}],$$

and

$$P_\lambda^2 = -\partial_t^2 + \sum_{i=1}^{D-1} P_i^2, \quad (6)$$

with the backgrounds $A_0^{cl} = 0$.

3 Static potential

Here we calculate the potential between two D0-branes at rest. The classical solution which represents two D0-branes in distance r can be introduced as

$$\begin{aligned} X_1^{cl} &= \frac{1}{2} \begin{pmatrix} r & 0 \\ 0 & -r \end{pmatrix}, \quad X_0^{cl} = i\partial_t \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \\ A_0^{cl} &= X_i^{cl} = 0, \quad i = 2, \dots, D-1. \end{aligned} \quad (1)$$

So one finds

$$P_1 = \frac{r}{2} \otimes \Sigma_3, \quad P_0 = i\partial_t \otimes 1_4, \quad P_i = 0, \quad i = 2, \dots, D-1, \quad (2)$$

where Σ_3 is the adjoint representation of the third Pauli matrix, $\Sigma_3^* = [\sigma_3, *]$. The eigenvalues of Σ_3 are 0, 0, +2, -2.

The operator P_λ^2 will be found to be

$$P_\lambda^2 = -\partial_t^2 \otimes 1_4 + \frac{r^2}{4} \otimes \Sigma_3^2, \quad (3)$$

and the one-loop effective action can be computed

$$\begin{aligned} V(r) &= \left(\frac{D}{2} - 1\right) \text{Tr} \log(P_\lambda^2) \\ &= -2\left(\frac{D}{2} - 1\right) \int_0^\infty \frac{ds}{s} \int_{-\infty}^\infty dk_0 e^{-s(k_0^2 + r^2)} \\ &\quad + \text{traces independent of } r, \end{aligned} \quad (4)$$

where 2 is for the degeneracy in eigenvalue 4 of Σ_3^2 , and k_0 is for the eigenvalues of the operator $i\partial_t$. In writing the second line we have used

$$\ln\left(\frac{u}{v}\right) = \int_0^\infty \frac{ds}{s} (e^{-sv} - e^{-su}).$$

The integrations can be done and one finds

$$\begin{aligned} V(r) &= -2\left(\frac{D}{2} - 1\right) \int_0^\infty \frac{ds}{s} \left(\frac{\pi}{s}\right)^{\frac{1}{2}} e^{-sr^2} \\ &= 4\pi\left(\frac{D}{2} - 1\right) |r| + \infty \text{ independent of } r. \end{aligned} \quad (5)$$

The linear potential is the same of "cigar" model, see e.g. [17]. Also it is the same which is consistent with spin-mass Regge trajectories [18]. By restoring the α' the potential will be found to be

$$V(r) = 4\pi\left(\frac{D}{2} - 1\right) \frac{|r|}{2\pi\alpha'} \quad (6)$$

which has the dimension lenght^{-1} . By comparison with Regge model one can have an estimation for α' [18].

It is assumed that the above potential causes pair production when quark and anti-quark are separated sufficiently far. The minimum distance sufficient for pair production depends on α' and the mass of the lightest quark. So although the linear behavior of the potential is not for infinite separations, but absence of free quarks remains expectable.

Fast Decaying D0-branes:

For two *fast decaying* D0-branes one can again calculate the above potential. This work can be done by putting for example a Gauss function for k_0 in the (4). This work

is equivalent with restricting the eigenvalues of the operator $i\partial_t$, with this in mind that eigenvalues of operators $(X, i\partial_t, \dots)$ represent the information correspond to classical values of D0-branes ². So one finds

$$\begin{aligned} V(r) &= -2\left(\frac{D}{2} - 1\right) \int_0^\infty \frac{ds}{s} \int_{-\infty}^\infty dk_0 \left(\frac{1}{\Delta} e^{-\frac{(k_0 - T)^2}{\Delta^2}} \right) e^{-s(k_0^2 + r^2)} \\ &\sim r^\xi, \quad 0 < \xi < 1, \end{aligned} \quad (7)$$

a result consistent with the phenomenology of heavy quarks [18] which we know that their weak decay rate grows with $(mass)^5$. In the extreme limit $\Delta \rightarrow 0$ which the two D0-branes see each other "instantaneously" one can take them as two D(-1)-branes (: D-instantons). The dynamics of D(-1)-branes are described by the action (4) but instead of the taking X_0 as $i\partial_t$ one takes X_0 as a matrix which its eigenvalues represent the "time"s which D(-1)-branes occur. A classical solution as

$$X_1^{cl} = \frac{1}{2} \begin{pmatrix} r & 0 \\ 0 & -r \end{pmatrix}, \quad X_0^{cl} = \begin{pmatrix} t_0 & 0 \\ 0 & t_0 \end{pmatrix}, \quad A_0^{cl} = X_i^{cl} = 0, \quad i = 2, \dots, D-1, \quad (8)$$

represents two D(-1)-branes occurred in distance r and time t_0 . The potential can be obtained easily

$$V(r) \sim -2\left(\frac{D}{2} - 1\right) \int_0^\infty \frac{ds}{s} e^{-s(r^2)} \sim \ln r, \quad (9)$$

which is consistent with phenomenology [18].

4 Conclusion and Discussion

In this letter we calculate the effective potential between two D0-branes and the result appeared to be the known linear one. For two fast decaying quarks, which in the extreme limit see each other instantaneously we obtained potentials in agreement with the phenomenology. As further push of our calculations it is reasonable to calculate scattering amplitudes by these potentials.

Why non-commutativity?

Special relativity in a modern compact definition may be represented as follows:

A modification of space-time which to be prepared as a ground for the natural and theoretically consistent propagation of fields.

So one learns that the space-time makes a X_μ 4-vector which behaves like the gauge field A_μ 4-vector (spin 1) under the boost transformations.

Also in this way Supersymmetry(SUSY) is a natural continuation of the special relativity program: Including spin $\frac{1}{2}$ sectors to the coordinates of space-time which are corresponded to the fermions of the nature. This leads one to the space-time formulation of the SUSY theories.

²The eigenvalues of $i\partial_t$ are different from their quantum mechanical analogy which due to the Schrodinger's equation are energy.

Now, what may be modified if nature has non-Abelian (non-commutative) gauge fields? In present nature non-Abelian gauge fields can not make spatially long coherent states; they are confined or too heavy. But the picture may be changed inside a hadron or very near of an electron. In fact recent developments of string theories sound this change and it is understood that non-commutative coordinates and non-Abelian gauge fields are two sides of one coin, as is mentioned in Sec. 2. The future theoretical research in this area may make clear the relations.

References

- [1] J.H. Schwartz, "*Lectures on Superstring and M-Theory Dualities*", hep-th/9607201; C.V. Vafa, "*Lectures on String and Dualities*", hep-th/9702201; A. Sen, "*An Introduction to Nonperturbative String Theory*", hep-th/9802051; E. Kiritsis, "*An Introduction to Nonperturbative String Theory*", hep-th/9708130.
- [2] J. Polchinski, Phys. Rev. Lett. **75** (1995) 4724, hep-th/9510017.
- [3] J. Polchinski, "*Tasi Lectures on D-Branes*", hep-th/9611050.
- [4] J. Polchinski, "*Strings and QCD*", hep-th/9210045.
- [5] A.M. Polyakov, "*Gauge Fields and Strings*", Harwood Academic Publishers, Chur (1987).
- [6] A.M. Polyakov, Nucl. Phys. **B486** (1997) 23; F. Quevedo and C.A. Trugenberger, Nucl. Phys. **B501** (1997) 143; M.C. Diamantini and C.A. Trugenberger, "*Geometric Aspects of Confining Strings*", hep-th/9803046.
- [7] R. Peschanski, "*Is QCD at Small x a String Theory?*", hep-ph/9710483; "*Dual Shapiro-Virasoro Amplitudes in The QCD Dipole Picture*", Phys. Lett. **B409** (1997) 491, hep-ph/9704342; A. Bialas, H. Navelet and R. Peschanski, "*The QCD Triple Pomeron Coupling from String Amplitudes*", hep-ph/9711442.
- [8] F.E. Close, "*An Introduction to Quarks and Partons* ", Academic Press (1979).
- [9] A. Donnachie and P.V. Landshoff, Phys. Lett. **B296** (1992) 227.
- [10] E. Witten, Nucl. Phys. **B460** (1996) 335, hep-th/9510135.
- [11] W. Taylor, "*Lectures on D-Branes, Gauge Theory and M(atrices)*", hep-th/9801182; C. Gomes and R. Hernandez, "*Fields, Strings and Branes*", hep-th/9711102; R. Dijkgraaf, "*Les Houches Lectures on Fields, Strings and Duality*" , hep-th/9703136.
- [12] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, Phys. Rev. **D55** (1997) 5112, hep-th/9610043.

- [13] N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, Nucl. Phys. **B498** (1997) 467, hep-th/9612115.
- [14] T. Banks, "*Matrix Theory*", hep-th/9710231.
- [15] D. Kabat and P. Pouliot, Phys. Rev. Lett. **77** (1996) 1004, hep-th/9603127; U.H. Danielsson, G. Ferretti and B. Sundborg, Int. J. Mod. Phys. **A11** (1996) 5463, hep-th/9603081.
- [16] L.F. Abbot, Acta Phys. Polonica, **B13** (1982) 33.
- [17] P.D.B. Collins, A.D. Martin and E.J. Squires, "*Particle Physics and Cosmology*", John-Wiley and Sons (1989).
- [18] Fayyazuddin and Riazuddin, "*A Modern Introduction to Particle Physics*", World Scientific (1992).